2.6

Graphs of Rational Functions

Rational Functions:

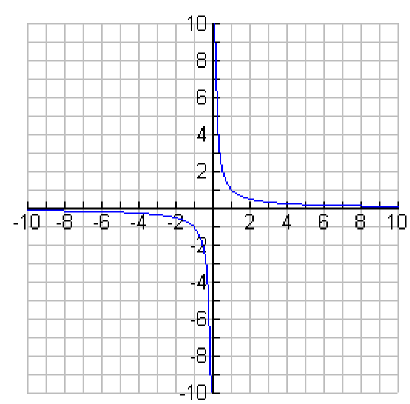
Let *f* and g be polynomial functions with g(x) ≠ 0. The function given by $r(x)=\frac{f(x)}{g(x)}$ is a rational function.

Finding the Domain of a Rational Function:

See example 1 pg. 237.

The Reciprocal Function:

f(x) = $\frac{1}{x}$



Domain: $(-\infty,0)\cup(0,\infty)$

Range: $(-\infty,0)\cup(0,\infty)$

Continuity: All x $\neq$0

Decreasing on $(-\infty,0)\ and\ (0,\infty)$

Symmetric with respect to the origin (odd)

Unbounded

No local extrema

Horizontal asymptote: y = 0, Vertical asymptote: x = 0

End Behavior: $\lim_{x\to-\infty} f(x)=0$ and $\lim_{x\to\infty} f(x)=0$

Transformations of the Reciprocal Function:

Remember the reciprocal function is y = $\frac{1}{x}$.

The graph of any rational function of the form $g(x)=\frac{ax+b}{cx+d}$ can be obtained through transformations of the reciprocal function. If the degree of the numerator is greater than or equal to the degree of the denominator, use polynomial division to rewrite the rational function.

See example 2 pg. 239.

Limits and Asymptotes

Graphical Features of a Rational Function

$$y=\frac{f(x)}{d(x)}=\frac{(a_n x^n+...)}{(b_m x^m+...)}$$

1. End behavior asymptote:

If *n* < *m*, the end behavior asymptote is the horizontal asymptote y = 0.

If *n* = *m*, the end behavior asymptote is the horizontal asymptote $y=\frac{a_n}{b_m}$.

If n > m, the end behavior asymptote is the quotient polynomial function *y = q(x),* where *f(x) = d(x)q(x) + r(x).* There is no horizontal asymptote. (**Slant asymptote**)

2. Vertical asymptotes: These occur at the zeros of the denominator provided that the zeros are not also zeros of the numerator or equal or greater multiplicity.

3. x-intercepts: These occur at the zeros of the numerator, which are not also zeros of the denominator .

4. y-intercept: This is the value of f(0), if defined.

See examples 4, 5, 6, 7 starting pg. 241